Frustrated total reflection: The double-prism revisited

A. Haibel,[1] G. Nimtz,[1] and A. A. Stahlhofen[2]
[1]*II. Physikalisches Institut, Universität zu Köln, Zülpicher Strasse 77, D-50937 Köln, Germany*
[2]*Institut für Physik, Universität Koblenz, Rheinau 1, D-56075 Koblenz, Germany*
(Received 4 May 2000; published 19 March 2001)

Geometrical optics prohibits any penetration of light into an optically rarer medium in the case of total reflection. When sandwiching, however, the rarer medium between optically denser media, a transmitted beam can be observed in the third medium. The experiment is often realized by a double-prism arrangement [1]; the effect is called frustrated total reflection due to the enforced transmission. Amazingly, the reflected and transmitted beams are shifted with respect to geometrical optics as conjectured by Newton [2] and experimentally confirmed by Goos-Hänchen 250 years later [3]. However, inconsistent results on the spatial shifts have been reported [4–7]. Here we report on measurements of the Goos-Hänchen shift in frustrated total reflection with microwaves. We found an unexpected influence of the beamwidth and angle of incidence on the shift. Our results are not in agreement with both previous experiments [6,7] and theoretical predictions [8–10]. The topic of frustrated total reflection is important for both fundamental research and applications [11–13].

DOI: 10.1103/PhysRevE.63.047601 PACS number(s): 42.25.Bs, 42.25.Gy, 73.40.Gk, 42.50.−p

When a beam of light hits an interface between different dielectrics with indices of refraction $n_1 > n_2$ under an angle $\theta_i > \theta_c := \arcsin n_2/n_1$, geometrical optics predicts total reflection of the incoming beam. In reality, however, the incoming beam penetrates into the second medium and travels for some distance parallel to the interface before being scattered back into the first medium (Fig. 1). The amazing shift of the reflected beam has been conjectured by Newton [2] and measured for the first time by Goos and Hänchen [3]. In the second medium the wave is characterized by the wave number $k_\parallel := k_0 n \sin\theta_i$ describing the propagation parallel to the interface and the imaginary $k_\perp := i k_0 \sqrt{n^2 \sin^2\theta_i - 1}$ associated with an instantaneous spread perpendicular to the interface and an exponential decay in this direction. ($k_0 = 2\pi/\lambda_0$ is the wave number in vacuum.)

If a third medium $n_3 = n_1$ is used to probe the ''evanescent'' wave in the second medium, the reflection becomes ''frustrated.'' Photonic tunnelling across the second medium to the third takes place as sketched in Fig. 1 for two prisms.

This realization of frustrated total reflection (FTR) has been used for the first time by Bose [1] to study the transmission of an incoming beam across the gap.

The Goos-Hänchen shift D is expected to be a delicate function of air gap, of polarization, of angle of incidence, and probably of beamwidth [6,7,14,15]. We measured the influence of all these parameters and compare our new results with previous experimental data and with theoretical predictions.

The Goos-Hänchen shift D is commonly derived from

$$D := \frac{\partial\varphi}{\partial k_\parallel}, \tag{1}$$

where $k_\parallel$ is the real wave number introduced above; φ is the phase shift of the beam [16]. This formula leads to equal shifts in reflection and transmission [6–10]. Here we report the results for the case of reflection, which actually are in agreement with those measured in transmission.

The experimental setup is shown in Fig. 2. The propagation time antenna-prism-antenna is longer than the signals half-width of 8 ns. The experimental setup provides a perfect asymptotic measurement, i.e., transmitter and sample are decoupled. Between the prism surface and the horn antenna a standing wave could not form as we checked by varying this distance up to half a wavelength.

First we investigated the polarization influence on the Goos-Hänchen shift. Previous measurements and theoretical predictions [6,7] reported a polarization dependence of this shift, with a magnitude about 90% larger in TM polarization than in TE polarization. For large beamwidths (diameter 190 mm and 350 mm) and $\theta_i = 45°$, the Goos-Hänchen shift reaches a constant asymptotic value with increasing the air gap, which equals roughly the wavelength λ_0. The TE polarization results are in agreement with the theoretical expectations [6,8–10]. But comparing the measured data of TE

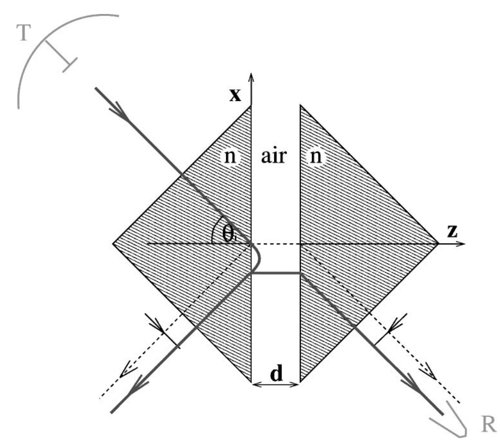

FIG. 1. Sketch of the experimental apparatus showing the parabolic transmitting antenna (T), the prisms, the air gap of width d, the horn antenna used as receiver (R), and the symmetrical shifts of the reflected/transmitted beams, where $\theta_i > \theta_c = \arcsin 1/n$ is the angle of incidence. θ_c is the critical angle of total reflection. The shift of the evanescent wave parallel to the surface in air represents the Goos-Hänchen shift D.

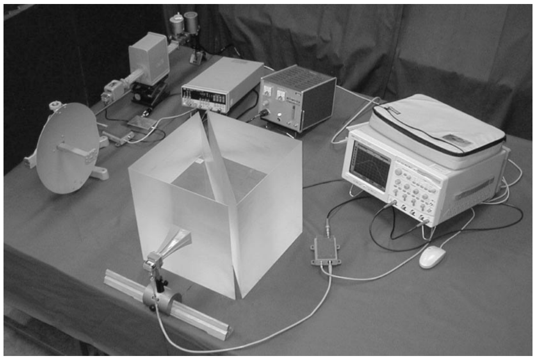

FIG. 2. A picture of the experiment. The prisms, cut from a cube of perspex with a side length of 400 mm, have an index of refraction $n=1.605$ ($\hat{=}\theta_c=38.5°$) at the frequency in question (9.15 GHz). Microwaves with $\lambda_0=32.8$ mm, generated in a klystron (2K25) are fed into a parabolic transmitter antenna guaranteeing quasiparallel beams. The beam spreading is less than 2° as follows from $\sin\phi=\lambda_0/2bn$ with diameter $b_{\mathrm{antenna}}=350$ mm and all beam components are in the range of total reflection. This was verified by measuring the transmission damping depending on the air gap between 0 and 50 mm. The damping would be 1.8 dB in the case of normal reflection compared with our measured 36 dB for the case of $\theta_i=45°$ and a 50-mm gap. The measured value of 7.2 dB/10 mm is in agreement with the theoretical transmission Ref. [17]. The signals have been picked up by a microwave horn and fed across an amplifier to an oscilloscope (HP 54825A). A metallic reflector placed at the base of the first prism to determine the position of the reflected beam in the case of geometrical optics. The results presented here are averaged values of several runs with error bars. (For the photo we put the various components near together to present all of them in one picture.)

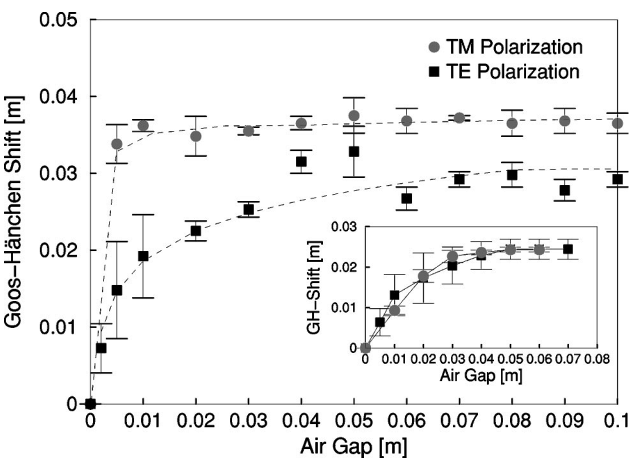

FIG. 3. The Goos-Hänchen shift vs air gap for TE and TM polarizations for a small beam (aperture 80 mm): the shift measured for a TM-polarized beam is obviously larger than the shift for a TE-polarized beam qualitative but not quantitative in agreement to previous experiments [6,7]. The 60-mm beam not included in the figure behaves in the same way. The insert shows the Goos-Hänchen shift for a TM- and a TE-polarized beam using a larger aperture (190 mm). The plotted values are the shifts of the reflected beam by scanning the leg of the prism.

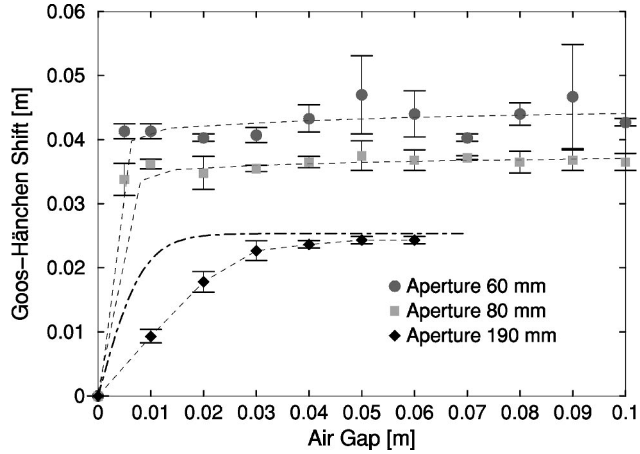

FIG. 4. The Goos-Hänchen shift vs air gap for different beam diameters in TM polarization and for $\theta_i=45°$: the shift measured for the large beams (no aperture or an aperture of 190 mm) is roughly in agreement with theoretical prediction (dot-dashed line) [8], while decreasing beam diameters lead to increasing shifts reaching the constant asymptotic value already for very small values of the air gap. The zero point was obtained by substituting the air gap with a metallic plate.

and TM polarization, both the predicted and previously measured polarization dependence of the beam shift was not observed (see the insert of Fig. 3). For smaller beams (diameter 60 mm and 80 mm) the shift of the TM-polarized beam was only about 20% larger than the shift of the TE-polarized beam (see Fig. 3). Obviously, deviations between TM- and the TE-polarized beams disappear for large beamwidths. The results show, that the width of the incoming beam with respect to the wavelength λ_0 is an important parameter for the Goos-Hänchen shift in FTR, a property which was neglected in theoretical discussions [8–10] so far.

Therefore we have investigated the influence of the beamwidth on the Goos-Hänchen shift at the angle of incidence

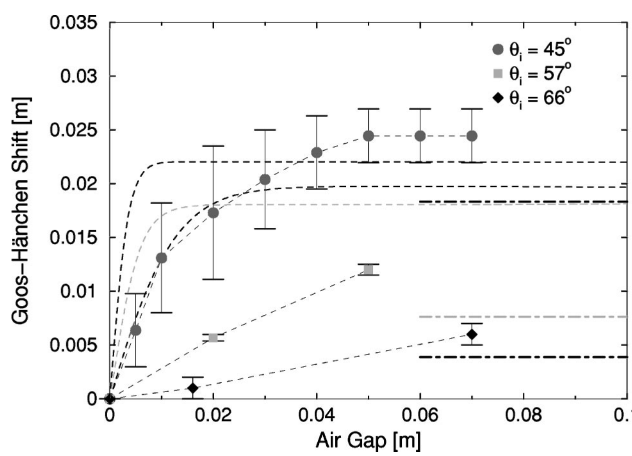

FIG. 5. The Goos-Hänchen shift vs air gap for different angles of incidence (aperture 190 mm, TE polarization). (The displacement of the shift on the leg of the prism is displayed.) The dot-dashed lines represent the asymptotic values of the calculated shift according to Renard's formula [20]. The dashed lines show the calculated data of Ghatak [10].

$\theta_i = 45°$. This experiment was carried out by limiting the diameter of the broad incoming beam ($b_{\mathrm{antenna}} = 350$ mm) with three circular apertures (diameters 190 mm, 80 mm, and 60 mm). The results are summarized in Fig. 4 and clearly demonstrate the influence of the beamwidth on the shift of a TM-polarized beam. (A TE-polarized beam produced the same results.) We found a totally different behavior than predicted before [6,14,15,18]. The Goos-Hänchen shift increases with decreasing beam dimension. Moreover, the smaller beams reach the constant asymptotic value already for very small air gaps. The beamwidth has been taken into account in the vicinity of the critical angle only [6,14,15,18]; for large angles of incidence as used here the classical results of Artmann [19] neglecting the beam width are recovered.

Diffraction effects cannot be in charge of the pronounced Goos-Hänchen shift depending on beam size. The form of the reflected or transmitted beam was unchanged and the observed diffraction pattern did not show significant side lobes within the Goos-Hänchen shift.

Another finding: The beam shift is a sensitive function for large angles of incidence. For angles far away from the critical angle inconsistent theoretical predictions for the Goos-Hänchen shift exist [4–6,8–10,20]. Former FTR studies were performed at angles of incidence near the critical angle only [4,6,7]. Our results for large angles of incidence revealed a strong decrease of the Goos-Hänchen shift with increasing angles (see Fig. 5).

For $\theta_i = 45°$, the measured shift is in agreement with theoretical predictions [6,8–10]. However, for $\theta_i = 57°$ the shift decreases about 50% and for $\theta_i = 66°$ more than 75%. The theoretical predictions [8,10], however, differ for large angles and even display the opposite dependence on θ_i. The asymptotic values for the Goos-Hänchen shift ($d \to \infty$) by comparison exhibit the observed behavior when derived from Renard's Eq. (5) [20].

The dependence of the beam shift in FTR on large angles of incidence, especially for the case of small gaps (strong frustration) obviously cannot be accounted for by current models.

We carried out a comprehensive study of the Goos-Hänchen shift vs the air gap as a function of polarization, beamwidth, and angle of incidence in an asymptotic experiment with microwaves. Microwaves behave like infrared or optical waves, can be used to investigate optical problems, and are easier to handle. The beamwidth was identified as an important parameter for the shift never minded before away from the critical angle. In addition we measured for the first time the Goos-Hänchen shift in FTR as a function of large angles of incidence. The experiments revealed a strong shift decrease with increasing angle of incidence. Most of the observations presented here contradict the theoretical descriptions and experimental studies of this historical problem, the Goos-Hänchen shift conjectured already 300 years ago by Newton [2,4,5,21–23]. Our results challenge current descriptions of the Goos-Hänchen shift in FTR and its applications in both fundamental and applied research [11–13].

[1] J. Ch. Bose, in *Collected Physical Papers of Sir Jagadis Chunder Bose* (Longmans, London, 1927), p. 42.
[2] I. Newton, *Opticks* (Dover, New York, 1952).
[3] F. Goos and H. Hänchen, Ann. Phys. (Leipzig) **1**, 333 (1947).
[4] V. Akylas, J. Kaur, and T. M. Knasel, Am. J. Phys. **44**, 77 (1976).
[5] D. J. Rhodes and C. K. Carniglia, J. Opt. Soc. Am. **67**, 679 (1977).
[6] J. J. Cowan and B. Anicin, J. Opt. Soc. Am. **67**, 1307 (1977).
[7] Ph. Balcou and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997).
[8] J. L. Agudin and A. M. Platzeck, J. Opt. (Paris) **9**, 187 (1987).
[9] J. Strnad and A. Kodre, Int. J. Theor. Phys. **9**, 393 (1974).
[10] A. S. Ghatak, M. R. Shenoy, I. C. Goyal, and K. Thyagarajan, Opt. Commun. **56**, 313 (1986).
[11] S. Zhu, A. W. Yu, D. Hawley, and R. Roy, Am. J. Phys. **54**, 601 (1986).
[12] B. Hecht, H. Bielefeldt, Y. Inouye, D. W. Pohl, and L. Novotny, J. Appl. Phys. **81**, 2492 (1997).
[13] M.-P. Marco and D. Barcelo, Meas. Sci. Technol. **7**, 1547 (1996).
[14] B. R. Horowitz and T. Tamir, J. Opt. Soc. Am. **61**, 586 (1971).
[15] H. Lai, F. C. Cheng, and W. K. Tang, J. Opt. Soc. Am. **3**, 550 (1986).
[16] C. K. Carniglia and L. Mandel, J. Opt. Soc. Am. **61**, 1035 (1971).
[17] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, New York, 1964), Vol. 2, p. 33.
[18] A. Puri and J. L. Birman, J. Opt. Soc. Am. A **3**, 543 (1986).
[19] K. Artmann, Ann. Phys. (Leipzig) **2**, 87 (1948).
[20] R. H. Renard, J. Opt. Soc. Am. **54**, 1190 (1964).
[21] M. Green, P. Kirkby, and R. S. Timsit, Phys. Lett. A **45**, 259 (1973).
[22] F. Bretenaker, A. Le Floch, and L. Dutriaux, Phys. Rev. Lett. **68**, 931 (1992).
[23] L. Dutriaux, Ph. Balcou, F. Bretenaker, and A. Le Floch, Europhys. Lett. **35**, 359 (1996).